Abstract Book

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Spectral Structure of the Neumann-Poincaré Operator on Thin Domains

Kazunori Ando (Ehime University, ando@cs.ehime-u.ac.jp)

We consider the spectral structure of the NeumannPoincar operators defined on the boundaries of thin domains in two and three dimensions. In two dimensions, we consider rectangle-shaped domains. We prove that as the aspect ratio of the domains tends to ∞, or equivalently, as the domains get thinner, the spectra of the NeumannPoincar operators are densely distributed in $[-1/2, 1/2]$. In three dimensions, we consider two different kinds of thin domains: thin oblate domains and thin cylinders. We show that in the first case the spectra are distributed densely in the interval $[-1/2, 1/2]$ as as the domains get thinner. In the second case, as a partial result, we show that the spectra are distributed densely in the half interval $[0, 1/2]$ as the domains get thinner. This talk is based on a joint work with H. Kang (Inha University, Korea) and Y. Miyanishi (Shinsyu University, Japan).

CT Reconstruction With Deep Learning: Uncertainty Quantification for the DIP

Riccardo Barbano (University College London, riccardo.barbano.19@ucl.ac.uk)

Existing deep-learning based tomographic image reconstruction methods do not provide accurate estimates of reconstruction uncertainty, hindering their real-world deployment. In this talk we present a method, termed as the linearised deep image prior (DIP) that estimates the uncertainty associated with reconstructions produced by the DIP with total variation regularisation (TV). We discuss how to endow the DIP with conjugate Gaussian-linear model type error-bars computed from a local linearisation of the neural network around its optimised parameters. This approach provides pixel-wise uncertainty estimates and a marginal likelihood objective for hyperparameter optimisation. Throughout the talk, we demonstrate the method on synthetic data and real-measured high-resolution 2D CT data, and show that it provides superior calibration of uncertainty estimates relative to previous probabilistic formulations of the DIP.

IFF: A Super-resolution Algorithm for Multiple Measurements

Zetao Fei (Hong Kong University of Science and Technology, zfei@connect.ust.hk)

The problem of reconstructing one-dimensional point sources from their Fourier measurements in a bounded interval $[-\Omega, \Omega]$ is known to be challenging in the regime where the spacing of the sources is below the Rayleigh length Ω/π . In this talk, we present a super-resolution algorithm, called Iterative Focusing-localization and Filtering (IFF), to resolve closely spaced point sources from their multiple measurements that are obtained by using multiple unknown illumination patterns. The new proposed algorithm requires no prior information about the source numbers and allows for a subsampling strategy that can circumvent the computation of singular-value decomposition for large matrices as in the usual subspace methods. In the talk, we will also discuss the theoretical results of the methods behind the algorithm. The derived results imply a phase transition phenomenon.

Numerical results show that the algorithm can achieve a stable reconstruction for point sources with a minimum separation distance that is close to the theoretical limit as well as the phase transition phenomenon predicted by the theoretical analysis.

Discovering Low-Dimensional State Variables from High-Dimensional Observation Data: A Means to Model Complex Dynamical Systems

Kuang Huang (Columbia University, kh2862@columbia.edu)

Data-driven approaches are transforming dynamical system modeling, especially when there is a large amount of raw data available, such as video streams. Most data-driven methods for modeling dynamical systems assume that observed data streams already correspond to a set of low-dimensional state variables that give a complete and non-redundant description of the system. However, despite the prevalence of computing power and AI, the process of identifying a set of state variables themselves from high-dimensional observation data has resisted automation. We propose a framework for determining how many state variables an observed system is likely to have, and what these variables might be, directly from very high-dimensional video streams. We also demonstrate the effectiveness of this approach using video recordings of a variety of dynamical systems, ranging from swinging pendulums to fire flames. Without prior knowledge of the underlying physics, our algorithm discovers candidate sets of state variables and produces stable long-term predictions of the system dynamics.

A Phase Field Approach to an Inverse Problem Governed by a Quasilinear Maxwell System

Andrew Kei Fong Lam (Hong Kong Baptist University, akflam@hkbu.edu.hk)

We tackle an inverse problem of reconstructing a discontinuous coefficient in quasilinear H(curl) magnetostatic equations from measurements in a subdomain. To overcome the ill-posedness of the inverse problem, we investigate two regularisations posed as constrained minimisation problems. The first involves perimeter penalisation and the second involves phase field regularisation. Existence of minimisers and consistency as the penalisation parameters tending to zero are discussed. We show under ideal situations a relation between parameters that allows one to directly obtain a solution to the inverse problem from the phase field solutions. Then, we investigate the convergence of the first order optimality conditions of the phase field problem to the optimal conditions obtained from shape calculus, leading to a necessary optimality system for the perimeter regularised problem. This is joint work with Irwin Yousept.

Stochastic Gradient Descent: Algorithmic Stability and Implicit Regularization

Yunwen Lei (Hong Kong Baptist University, yunwen@hkbu.edu.hk)

Stochastic gradient descent (SGD) has become the workhorse behind many machine learning problems. An important problem regarding SGD is how the models produced by SGD would generalize to testing examples. In this talk, I will present our recent results on the generalization analysis of SGD from the perspective of algorithmic stability. We introduce a new algorithmic stability concept to relax the existing restrictive assumptions and to improve the existing generalization bounds. Our results show how the implicit regularization can be achieved by tuning the step size and the number of passes.

CT Reconstruction with Deep Learning: an Overview of Approaches and Extensions for the DIP

Johannes Leuschner (University of Bremen, jleuschn@uni-bremen.de)

Computed tomography (CT) is an important tool in both medicine and industry. Over the past years, deep learning (DL) has been researched intensively with applications in various inverse imaging tasks including CT. We first give an overview of different DL approaches for CT, and compare a selection of methods qualitatively and quantitatively on synthetic, medical and industrial datasets. The deep image prior (DIP) is an unsupervised DL framework that in contrast to most DL approaches does not rely on a large training dataset, but only on the single degraded observation. We experiment with multiple extensions for DIP applied to CT: (i) total variation regularization can stabilize and improve the reconstruction, (ii) pretraining on a post-processing task with easy-to-generate synthetic data can significantly accelerate the slow DIP reconstruction, and (iii) restricting the optimization to subspaces defined by pretraining checkpoints allows for second order optimization and can alleviate overfitting. We demonstrate the effectiveness of the proposed extensions of DIP on different CT datasets, including simulated medical and real-measured ?CT data.

Minnaert Resonances for Bubbles in Soft Elastic Materials

Hongjie Li (Chinese University of Hong Kong, hjli@math.cuhk.edu.hk)

In this talk, the low-frequency resonance for acoustic bubbles embedded in soft elastic materials is discussed. This is a hybrid physical process that couples the acoustic and elastic wave propagations. By delicately and subtly balancing the acoustic and elastic parameters as well as the geometry of the bubble, we show that Minnaert resonance can occur for rather general constructions. This study poses a great potential for the effective realisation of negative elastic materials by using bubbly elastic media.

Inverse Random Potential Scattering for Elastic Waves

Jianliang Li (Hunan Normal University, lijl@amss.ac.cn)

This talk is concerned with the inverse elastic scattering problem for a random potential in three dimensions. Interpreted as a distribution, the potential is assumed to be a microlocally isotropic Gaussian random field whose covariance operator is a classical pseudo-differential operator. Given the potential, the direct scattering problem is shown to be well-posed in the sense of distributions by studying the equivalent Lippmann–Schwinger integral equation. For the inverse scattering problem, we demonstrate that the microlocal strength of the random potential can be uniquely determined with probability one by a single realization of the high frequency limit of the averaged compressional or shear backscattered far-field pattern of the scattered wave. The analysis employs the integral operator theory, the Born approximation in the high frequency regime, the microlocal analysis for the Fourier integral operators, and the ergodicity of the wave field.

Inverse Source Problems of Local, Nonlocal and Nonlinear Equations

Yi-Hsuan Lin (National Yang Ming Chiao Tung University, yihsuanlin3@gmail.com)

In this talk, we perform inverse source problems for local, nonlocal and nonlinear equations. Unlike linear differential equations, which always has gauge invariance. We investigate the gauge symmetry could be broken for several nonlinear and nonlocal equations, which leads unique determination results for certain equations. The talk is based on several joint works with Yavar Kian, Tony Liimatainen and Hongyu Liu.

Identification of the Spherical Shell Structured Sources from the Far Field Patterns with at Most Two Frequencies

Xiaodong Liu (Chinese Academy of Sciences, xdliu@amt.ac.cn)

It is well known that the far field patterns at finitely many frequencies are not enough to uniquely determine a general source. To establish uniqueness, we consider the acoustic scattering of spherical shell structured sources. The inverse source problem can then be thought as a parameter estimation problem of model-based inversion problem. However, without the use of additional constraints on the sources, there would be no hope to uniquely determine the inner diameters, the outer diameters and the scattering strengths from the measurements at a fixed frequency. Numerically, the classical iterative methods can not be applied directly because the number of the spherical shell structured sources is not known in advance. Besides, the iterative methods often depend on a fine initial guess, which is not practical in many applications. Moreover, the nonlinearity and illposedness of the inverse problem brings great challenge for the corresponding numerical simulations. We show that the number and the centers (i.e., locations) of the spherical shells can be uniquely determined by the far field patterns at a fixed frequency. Furthermore, we show that the scattering strengths, the inner and outer diameters can be uniquely determined by the far field patterns at two properly chosen frequencies. A non-iterative migration series method is designed to located the centers. The numerical simulations show that the reconstruction quality is the same as the direct sampling method. Finally, a convergence iterative method is designed for computing the inner diameters and outer diameters. An important feature of the proposed iterative method is that it avoids computing any derivatives. Some numerical examples are presented to verify the effectiveness and robustness of the proposed numerical scheme.

Determining a Parabolic System by its Boundary Data with Biological Applications

Catharine Wing Kwan Lo (City University of Hong Kong, wingkclo@cityu.edu.hk)

We consider the inverse problem of determining some coefficients of the nonlinear terms of a coupled system and discuss its biological applications

Inverse Scattering by Corners and Regular Transmission Eigenfunctions

Chun-Hsiang Tsou (National Central University, chtsou@math.ncu.edu.tw)

This work concerns on the recovery of polygonal shaped scatterers using a single far-field pattern measurement. At the same time, we studied the associated transmission eigenvalue problem and derived the regularity results on the transmission eignefunctions. On the side of inverse problems, we have applied the micro-local analysis and the quantitative unique continuation property. One of our original initiative is to use the singular decomposition of the solutions to elliptic equations near polygonal corners. We finally derived a double logarithmic type stability estimate to the inverse problem and an Holder type regularity to the associate transmission eigenfunctions. This a joint work with Prof. Hongyu Liu at City University HK.

Asymptotically Sharp Upper Bound for the Column Subset Selection Problem

Zili Xu (Hong Kong University of Science and Technology, xuzili@ust.hk)

The column subset selection has gained significant attention recently due to its wide applications in machine learning, scientific computing, and signal processing. It refers to the task of selecting exactly k columns of an input marix A that minimize the approximation error, i.e., the spectral norm of the residual matrix after projecting A onto the space spanned by the selected columns. In this talk we introduce an asymptotically sharp upper bound on the minimal approximation error obtained by using the method of interlacing polynomials . We present a deterministic polynomial-time algorithm that achieves this error bound (up to a computational error). Furthermore, we show that the machinery of interlacing polynomial also works for a column partition problem, which involves partitioning the columns of A into r subsets such that A can be well approximated by the columns from several different subsets. We show that the columns of a rank-d matrix A can be partitioned into r subsets S_1,..., S_r, such that the column space of A can be well approximated by the span of the columns in the complement of S_i for each i.

Stability and Uniqueness for Inverse Problems for Partial Differential Equations by Carleman Estimates

Masahiro YAMAMOTO (The University of Tokyo, myama@next.odn.ne.jp)

In the inverse problem, the numerical reconstruction is important. However, since inverse problems are mostly ill-posed, we have to clarify stability and uniqueness for sane numerical analysis. I make self-contained accounts for one main method based on Carleman estimates which establishes the stability and the uniqueness. Moreover I explain how resulting stability is useful for guaranteeing qualities of Tikhonovs regularization.

Subwavelength Localized Modes for Acoustic Waves

Sanghyeon Yu (Korea University, sanghyeon_yu@korea.ac.kr)

The recent development of subwavelength photonic and phononic crystals shows the possibility of controlling wave propagation at deep subwavelength scales. Subwavelength bandgap phononic crystals are typically created using a periodic arrangement of subwavelength resonators, for example, small gas bubbles in a liquid. In this talk, we consider various structures generated by modifying periodic phononic crystals by 'defects'. Our aim is to prove that the defect acts as a waveguide; waves of certain frequencies will be localized to, and guided. The key result is an original formula for the frequencies of the defect modes. This talk is based on joint works with Habib Ammari (ETH) and Erik Orvehed Hiltunen (Yale).

On Inverse Boundary Value Problems for Mean Field Games

Shen Zhang (City University of Hong Kong, szhang347-c@my.cityu.edu.hk)

We consider several inverse problems for mean field games in a bounded domain. By suitable boundary measurement, one can recover the running cost and Hamiltonian in mean field game system simultaneously if the running cost is nonlocally dependent on the measure.

A New Framework to Quantify the Uncertainty in Inverse Problems

Dr. Wenlong Zhang (Southern of Science and Technology, zhangwl@sustech.edu.cn)

In this talk, we investigate the regularized solutions and their finite element solutions to the inverse problems governed by partial differential equations, and establish the stochastic convergence and optimal finite element convergence rates of these solutions, under point wise measurement data with random noise. The regularization error estimates and the finite element error estimates are derived with explicit dependence on the noise level, regularization parameter, mesh size, and time step size, which can guide practical choices among these key parameters in real applications. The error estimates also suggest an iterative algorithm for determining an optimal regularization parameter.

Stochastic Asymptotical Regularization for Inverse Problems

Ye Zhang (Shenzhen MSU-BIT, ye.zhang@smbu.edu.cn)

We introduce Stochastic Asymptotical Regularization (SAR) methods for the uncertainty quantification of the stable approximate solution of ill-posed linear-operator equations, which are deterministic models for numerous inverse problems in science and engineering. We prove the regularizing properties of SAR with regard to mean-square convergence. We also show that SAR is an optimal-order regularization method for linear ill-posed problems provided that the terminating time of SAR is chosen according to the smoothness of the solution. A toy example and a real-world problem of biosensor tomography are studied to show the accuracy and the advantages of SAR: compared with the conventional deterministic regularization approaches for deterministic inverse problems, SAR can provide the uncertainty quantification of the quantity of interest, which can in turn be used to reveal and explicate the hidden information about real-world problems, usually obscured by the incomplete mathematical modeling and the ascendence of complex-structured noise.

Numerical Analysis of Diffusion Coefficient Identification for Elliptic and Parabolic Problems

Zhi Zhou (Hong Kong Polytechnic University, zhizhou@polyu.edu.hk)

Parameter identifications for differential equations represent a wide class of inverse problems. Conventionally, this class of inverse problems could be solved via an optimization approach, which is then discretized (using finite difference method, finite element methods, or deep neural networks) for practical implementation. Then one important issue is to derive a priori error estimates for the numerical reconstruction of the desired parameter. In this talk, we present our recent efforts to derive convergence rates for discrete schemes for recovering a spatially dependent diffusion coefficient in an elliptic or parabolic type problem, by suitably exploiting relevant stability results.