Some Open Inverse Problems

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I will describe some open problems in the theory of inverse problems.

An Inverse Problem of Recovering Density of an Elastic Medium

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Here we present an inverse problem in elastodynamics of recovering the density of a medium (locally) near its boundary. As a model problem we study the isotropic elasticity operator on a 3-dimensional bounded domain. Measuring the corresponding Dirichlet to Neumann map on a part of the boundary, we show the uniqueness of the density in a suitable neighborhood of the part of the boundary where the measurements have been taken.

On the geometric structures of conductive transmission eigenfunctions and their application

Xinlin Cao

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We are concerned with the intrinsic geometric structures of conductive transmission eigenfunctions. The geometric properties of interior transmission eigenfunctions were first studied by E. Bl?sten and H. Liu. It is shown in two scenarios that the interior transmission eigenfunction must be locally vanishing near a corner of the domain with an interior angle less than π. We significantly extend and generalize those results in several aspects. First, we consider the conductive transmission eigenfunctions which include the interior transmission eigenfunctions as a special case. Second, the vanishing property of the conductive transmission eigenfunctions is established for any corner as long as its interior angle is not π. That means, as long as the corner singularity is not degenerate, the vanishing property holds. Third, the regularity requirements on the interior transmission eigenfunctions are significantly relaxed in the present study for the conductive transmission eigenfunctions. In order to establish the geometric properties for the conductive transmission eigenfunctions, we develop technically new methods and the corresponding analysis is much more complicated. Finally, as an interesting and practical application of the obtained geometric results, we establish a unique recovery result for the inverse problem associated with the transverse electromagnetic scattering by a single far-field measurement in simultaneously determining a polygonal conductive obstacle and its surface conductive parameter.

Reconstruction of a locally perturbed planar surface using plasmonic sensing

Doosung Choi

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In this presentation, we provide a mathematical framework to explain physical mechanisms for the diffraction limit in near-field optics. We analyze the behavior of light under the influence of plasmonics which allows nano-sensing of samples. We consider a plasmonic nanoparticle placed near a locally perturbed planar surface. We use the Fourier analysis technique to image the sample from the plasmonic resonances due to the coupling between the nanoparticle and the perturbed planar surface. The main idea is to use a Möbius transformation and to express the far-field and the plasmonic resonances in terms of the Fourier coefficients of the transformed domain. We will show the images reconstructed from the contracted generalized polarization tensors of the transformed domain.

Localized resonance and its applications in imaging and cloaking

Youjun Deng

Central South University

In this talk, we shall introduce the notions of localized resonaces. Usually these problems are related to the spectral of the Neumann-Poincare operaters in different models. As an example, we provide a mathematical framework for localized plasmon resonance of nanoparticles. Using layer potential techniques associated with the full Maxwell equations, we derive small-volume expansions for the electro-magnetic fields, which are uniformly valid with respect to the nanoparticle's bulk electron relaxation rate. Then, we discuss the scattering and absorption enhancements by plasmon resonant nanoparticles. Finally, we show the applications of localiezed resonance in imaging and cloaking.

The monotonicity based method for the inverse crack scattering problem

Takashi Furuya

Nagoya University

The monotonicity based method for the inverse acoustic scattering problem is to understand the inclusion relation between an unknown object and an artificial one by comparing the far field operator and the artificial operator. In this talk, we develop this method to the inverse crack scattering problem. Our aim is to give the following two indicators: One is to determine whether an artificial small curve is contained in the unknown crack. The other one is whether an artificial large domain contain the unknown one.

Inverse problem for fractional Schrodinger equation with non-local and local boundary

Tuhin Ghosh

The HongKong Universtiy of Science and Technolodge

We will be discussing the fractional Calderon problem, where one tries to determine an unknown potential in a fractional Schrodinger equation from the exterior or boundary measurements of the solutions.

Reflection principle with applications to inverse obstacle scattering with a single incoming wave

Guanghui Hu

Beijing Computational Science Research Center

In this talk we will review the reflection principle for some partial differential equations, including the harmonic equation, bi-harmonic equation, Helmholtz equation and Navier equation etc.. The reflection principle gives a non-local extension formula for these PDEs across a hyper-plane under certain boundary condition. We will show how to apply a reflection principle to the determination of an elastic rigid obstacle of polyhedral type with a single incoming wave. This is a joint work with Johannes Elschner.

On inverse scattering for Stark Hamiltonians

Atsuhide Ishida

Tokyo University of Science

We consider one of the multidimensional inverse scattering problems for quantum systems governed by the Stark Hamiltonians.By applying the time-dependent method developed by Enss and Weder in 1995, we report that the high-velocity limit of the scattering operator determines uniquely the short-range interaction potentials. Moreover, we report that, when a long-range interaction potential is given, the high-velocity limit of the Dollard-type modified scattering operator determines uniquely the short-range part of the interactions. We allow the potential functions to belong to very broad classes. These results are improvements on the previous results obtained by Adachi and Maehara in 2007 and Adachi, Fujiwara, and Ishida in 2013.

Artifact suppression in X-ray CT images

Soomin Jeon

Korea Advanded Institute of Science and Technology

X-ray CT is one of the most powerful techniques for visualizing internal structure of an scanned object. Recently, it has been widely used in the field of industry as well as in the field of medicine. However, there are various kinds of artifact causing severe degradation of CT images. Therefore, developing an accurate and efficient artifact suppression method is becoming an important issue in X-ray CT imaging and its applications. In this talk, we will look around the mathematics in X-ray CT and some artifacts in CT images. Moreover, we will deal with methods to suppress them especially focusing on beam-hardening artifact due to metallic objects.

Corner Scattering

Jingni Xiao

The State University of New Jersey

We consider time-harmonic acoustic or electromagnetic scattering by penetrable inhomogeneous media. We examine the effect of nontrivial scattering due to the appearance of corners at the support of the inhomogeneities. Uniqueness of shape determination from one single measurement will be discussed as well.Joint with Blåsten Emilia, Fioralba Cakoni and Hongyu Liu.

Regularization of inverse problems by two-point gradient methods with convex constraints

Min Zhong

Southeast University

In this paper, we propose and analyze a two-point gradient method for solving inverse problems in Banace spaces which is based on the Landweber iteration and an extrapolation strategy. The method allows to use non-smooth penalty terms,including the L^1 and the total variation-like penalty functionals, which are significant in reconstructing special features of solutions such as sparsity and piecewise constancy in practical applications. The design of the method involves the choices of the step sizes and the combination parameters which are carefully discussed. Numerical simulations are presented to illustrate the effectiveness of the proposed method.

Variational Bayes' approach for functions and applications to some inverse problems

Junxiong Jia
Xi'an Jiaotong University

Bayesian approach as a useful tool for quantifying uncertainties has been widely used for solving inverse problems of partial differential equations (IPPDE). One of the key difficulties for employing Bayesian approach is how to extract information from the posterior probability measure. Variational Bayes' method (VBM) is one of the most activate research topics in the field of machine learning, which has the ability to extract posterior information approximately by using much lower computational resources compared with the sampling type method. In this talk, we generalize the usual finite-dimensional VBM to infinite-dimensional space, which makes the usage of VBM for IPPDE rigorously. General infinite-dimensional mean-field approximation theory has been established, and has been applied to abstract linear inverse problems with Gaussian and Laplace noise assumption. Finally, two numerical examples are given which illustrate the effectiveness of the proposed approach.

On decay rate of solutions for the stationary Navier-Stokes equation in an exterior domain

Puzhao Kow
National Taiwan University

In this talk, we consider the asymptotic behavior of an incompressible fluid around a bounded obstacle.By adapting the Schauder's estimate for stationary Navier-Stokes equation to improve the regularity, the problem is solved by using appropriate Carleman estimates. It should be noted that the minimal decaying rate for a general scalar equation is $\exp(-C|x|^{2+})$. However,the structure of the Navier-Stokes equation is special. Under the assumption for any non-trivial solution to be uniform bounded which is weaker than those in [LUW11], we got the minimal decaying rate is $\exp(-C|x|^{\frac{3}{2}+})$ which is better than the results in general scalar cases [KL19].

References

[KL19] Pu-Zhao Kow, Ching-Lung Lin, On decay rate of solutions for the stationary Navier-Stokes equation in an exterior domain, J. Dierential Equations 266 (2019) 3279-3309.

[LUW11] Ching-Lung Lin, Gunther Uhlmann, Jenn-Nan Wang, Asymptotic behavior of solutions of the stationary Navier-Stokes equations in an exterior domain, Indiana Univ. Math. J. 60 (6) (2011) 2093-2106.

Weak Galerkin Method for Electrical Impedance Tomography Inverse Problem

Ying Liang

The Chinese University of Hong Kong

We propose a novel weak Galerkin method to solve the ill-posed elliptic Cauchy problem. We apply dual arguments to develop this stable algorithm for the Cauchy problem, with first order nonconforming approximation space. We derive the error estimates for this algorithm in L2 norm, and provide stability analysis with respect to the Dirichlet boundary condition. Numerical experiments are performed to validate the theoretical results.

Inverse problems for elliptic equations with power type nonlinearities

Yi-Hsuan Lin

University of Jyväskylä

We introduce a method for solving Calder\'on type inverse problems for semilinear equations with power type nonlinearities. The method is based on higher order linearizations, and it allows one to solve inverse problems for certain nonlinear equations in cases where the solution for a corresponding linear equation is not known. Assuming the knowledge of a nonlinear Dirichlet-to-Neumann map, we determine both a potential and a conformal manifold simultaneously in dimension 2, and a potential on transversally anisotropic manifolds in dimensions $n\geq 3$. In the Euclidean case, we show that one can solve the Calder\'on problem for certain semilinear equations in a surprisingly simple way without using complex geometrical optics solutions.

A Linearized Inverse Boundary Value Problem in Elasticity with Transversally Isotropic Perturbations

Yang Yang

Michgan State University

We consider a linearized inverse boundary value problem for the elasticity system.From the linearized Dirichlet-to-Neumann map at zero frequency, we show that a transversely isotropic perturbation of a homogeneous isotropic elastic tensor can be uniquely determined. From the linearized Dirichlet-to-Neumann map at two distinct positive frequencies, we show that a transversely isotropic perturbation of a homogeneous isotropic density can be identifi ed as well.

Analysis and Application of Optimal Transport For Challenging Seismic Inverse Problems

Yunan Yang

Courant Institute

In seismic exploration, sources and measurements of seismic waves on the surface are used to determine model parameters representing geophysical properties of the earth. Full-waveform inversion (FWI) is a nonlinear seismic inverse technique that inverts the model parameters by minimizing the difference between the synthetic data from the forward wave propagation and the observed true data in PDE-constrained optimization. The traditional least-squares method of measuring this difference suffers from three main drawbacks including local minima trapping, sensitivity to noise, and difficulties in reconstruction below reflecting layers. Unlike the local amplitude comparison in the least-squares method, the quadratic Wasserstein distance from the optimal transport theory considers both the amplitude differences and the phase mismatches when measuring data misfit. I will briefly review our earlier development and analysis of optimal transport-based inversion and include improvements, for example, a stronger convexity proof. The main focus will be on the third ``challenge'' with new results on sub-reflection recovery.

An inverse Sturm-Liouville problem revisited: inversion of trace formulas

Jian Zhai

The HongKong Universtiy of Science and Technolodge

The classical problem ``Can we hear the density of a string?" can be formulated as an inverse spectral problem for a Sturm-Liouville operator. We propose a new numerical scheme for the reconstruction of the density, based on inverting a sequence of trace formulas. Numerical experiments will be presented. Joint work with Xiang Xu.

Electromagnetic scattering and inverse scattering by rough surfaces and obstacles

Lei Zhang

Heilongjiang University

In this talk, we mainly focus on the electromagnetic scattering and inverse scattering problems with rough surfaces and obstacles which lie above (below) the surfaces. We develop a novel integral equation method for the scattering problem, the wellposeness of the scattering problem is proved, and then obtain the uniqueness results for the inverse scattering problem. Some theoretical results for the local stability will be addressed too.